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Experimental Investigations of the Vortex Flow on Delta Wings at High Incidence

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It is reported in the literature that strongly asymmetric vortex flow occurs on the lee side of slender delta wings at high incidence, similar to that observed on slender cones, and a boundary is given for the onset of asymmetry vs wing aspect ratio. To study this phenomenon, dye flow-visualization tests were carried out on two slender, sharp-edged delta wings of aspect ratio A = 0.56 and 0.28 up to high incidences in a water tunnel. Another wing with A = 0.56 was studied in a wind tunnel, using a smoke flow-visualization technique. Reynolds numbers based on the root chord in the water tunnel and wind tunnel were $Re_c = 3.4 \times 10^4$ and 1.32×10^5, respectively. No strongly asymmetric vortex flow was observed in our tests before vortex breakdown occurred on the wing. Subsequent evaluation of flow observations reported elsewhere did not reveal strongly asymmetric vortex flow on slender wings of various planforms. One delta wing reported to exhibit strongly asymmetric vortex configurations; on closer study, it probably had near its apex the shape of something like a thick, elliptic cone, rather than that of a flat, thin delta wing. Theoretical results for inviscid flow past slender cones at incidence strongly suggest that on a thin delta wing no strongly asymmetric vortex flow occurs of the type found on slender, circular, or thick elliptic cones.

Introduction and Objective

IT was reported in fairly early work that on slender delta wings in low-speed flow the initially symmetrical (with respect to the wing symmetry plane) leading-edge vortices on the lee side became strongly asymmetric at some critical angle of incidence, before vortex breakdown occurred.[1] This phenomenon has since been reported various times, e.g., see Refs. 2 and 3.

The present work was initially planned to study this asymmetry of voxtex flow on slender delta wings and to attempt to suppress the asymmetry be means of a fin on the lee side between the vortices in analogy to previous successful experiments with a fin on a slender, circular cone.[4,5] To this end, the flow past two very slender, sharp-edged delta wings was studied in a water tunnel up to high incidences at zero yaw. However, no asymmetric vortex-flow configuration was observed before vortex breakdown occurred on the wings.

The objective was then changed with the intention to verify these results in further, now wind tunnel tests and find indications as to the reasons for this obvious discrepancy with earlier results. Work quoted in support of the occurrence of strong asymmetry of vortex flow on delta wings was looked at again, resulting in different interpretations. In addition, results of an inviscid-flow theory reported recently[6] were consulted in search of an explanation. This theory provides indications as to the effect of change of cross-sectional thickness and shape on vortex-flow asymmetry for slender cones.

The investigations described here were largely carried out at the Deutsche Forschungsanstalt für Luft- und Raumfahrt (DLR), Göttingen, Germany, and partly at King Fahd University of Petroleum & Minerals (KFUPM), Dhahran, Saudi Arabia.

Testing Facilities, Models, and Experimental Techniques

The water tunnel of DLR that was used for part of the investigations has a closed circuit. The horizontal test section, with a free water surface, has a cross-sectional area of 0.25 × 0.33 m and length of 1.25 m. Maximum water velocity in the empty test section is $V_\infty \cong 0.5$ m/s. The tunnel is operated continuously. The pump is driven by a 4-kW electric motor.

The wind tunnel of KFUPM is of the open return type. The closed, horizontal test section has a cross-sectional area of 0.8 × 1.1 m and a length of 3 m. Maximum wind velocity in the empty test section is $V_\infty = 35$ m/s. The blower is driven by a 15-kW electric motor.

Two delta wing geometries with sharp edges everywhere and flat suction sides were studied: one with aspect ratio $A = 0.56$, i.e., leading-edge sweep angle $\varphi = 82$ deg, and one with $A = 0.28$, i.e., $\varphi = 86$ deg. Model lengths were $c = 200$ and 300 mm for the water tunnel and wind tunnel, respectively. The model geometries are shown in Figs. 1. The models in the water tunnel were supported by a sting from downstream. In the wind tunnel, the model was held on the pressure side by a small fin, which, in turn, was supported by a sting extending downstream.

In the water tunnel, a flow-visualization technique was used, where very small quantities of dye (sodium fluorescein) were applied to the model surface at appropriate locations, with the model outside the test section. After having placed the model into the water, the flow patterns immediately became visible and remained so, typically, for some minutes. They were recorded on standard color film.

In the wind tunnel, a smoke flow-visualization technique was used. The smoke was produced by passing oil through an electrically heated coil; it was introduced into the flow upstream of the model. The flow patterns were made visible in laser-light sheets arranged perpendicular to the wing axis. They were recorded on standard color film.

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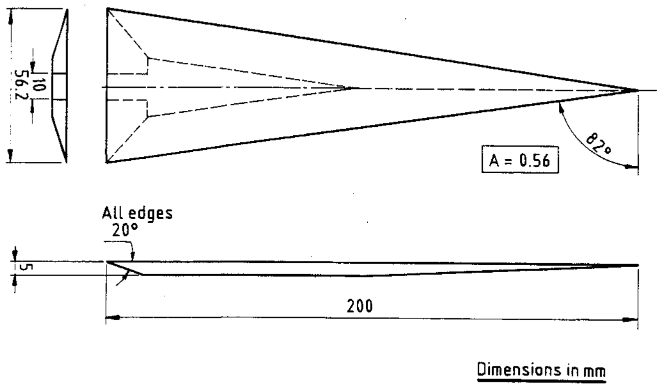


Fig. 1a Delta wing model, $A = 0.56$ (water tunnel).

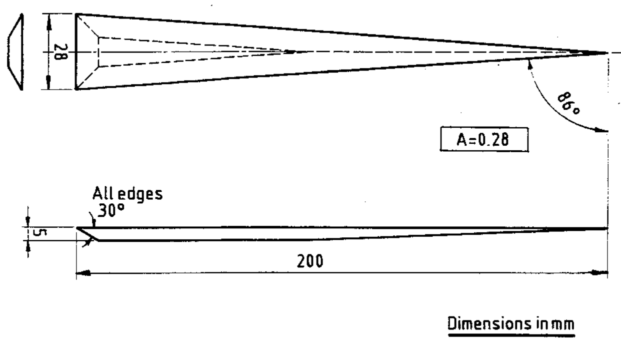


Fig. 1b Delta wing model, $A = 0.28$ (water tunnel).

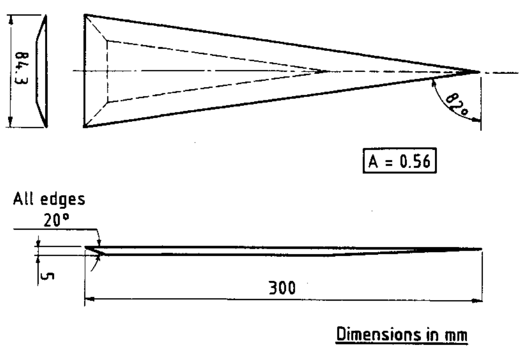


Fig. 1c Delta wing model, $A = 0.56$ (wind tunnel).

Testing Conditions

The experiments in the water tunnel were carried out on two delta wings, with aspect ratios $A = 0.56$ and 0.28 and a length of $c = 0.2$ m, at a freestream velocity $V_\infty = 0.17$ m/s. The Reynolds number was

$$Re_c = (V_\infty \cdot c)/\nu = 3.4 \times 10^4$$

and the range of angles of incidence α covered 15 deg $\leq \alpha \leq$ 39 deg. The model geometries are given in Figs. 1a and 1b.

The experiments in the wind tunnel were carried out on a delta wing of length $c = 0.3$ m and an aspect ratio of $A = 0.56$ at a freestream velocity $V_\infty = 6.6$ m/s. The Reynolds number was $Re_c = 1.32 \times 10^5$ and the incidence range covered 10 deg $\leq \alpha \leq$ 45 deg. Some tests were carried out at a yaw angle of $\beta = 6$ deg for $\alpha = 30$ and 35 deg. The light sheet was normal to the wing axis, at various chordwise locations. The model geometry is given in Fig. 1c.

Results and Discussion

Experimental and Theoretical Results

It has been reported in the literature that, on slender delta wings in low-speed flow, strong asymmetry of the lee-side vortex flow occurs at and above some critical angle of incidence α_{onset} before vortex breakdown is observed on the wing.[1] A boundary given for the onset of this vortex-flow asymmetry vs aspect ratio A presented in Ref. 1 was used subsequently without additional, substantiating results given.[2,3] The curve in the form used in Ref. 3 is given in Fig. 2. Such strongly asymmetric vortex flow also occurs on the lee side of slender, pointed noses at high angles of incidence. It was shown for a slender, circular cone in low-speed flow that placing a fin between the vortices suppressed the strong vortex-flow asymmetry in the incidence range investigated.[4,5] Based on this result, it was intended to explore the effect of a fin on the asymmetric vortex flow behind slender delta wings. To this end, two delta wings were selected on the basis of the boundary for onset of strong asymmetric vortex flow of Fig. 2, having leading-edge sweep angles $\varphi = 82$ and $\varphi = 86$ deg; i.e., aspect ratios $A = 0.56$ and 0.28, respectively. According to Fig. 2, onset of strong vortex-flow asymmetry was to be expected at $\alpha_{onset} \cong 24$ deg for leading-edge sweep $\varphi = 82$ deg and at $\alpha_{onset} \cong 16$ deg for $\varphi = 86$ deg.

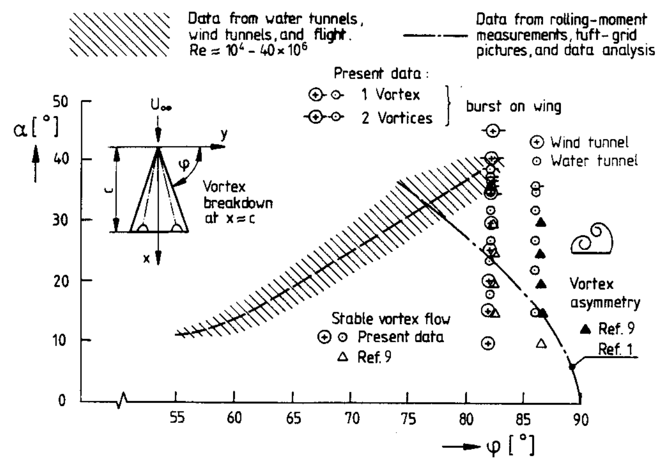


Fig. 2 Dependence on leading-edge sweep of onset angles of attack of vortex breakdown and asymmetry for delta wings in low-speed flow.[3]

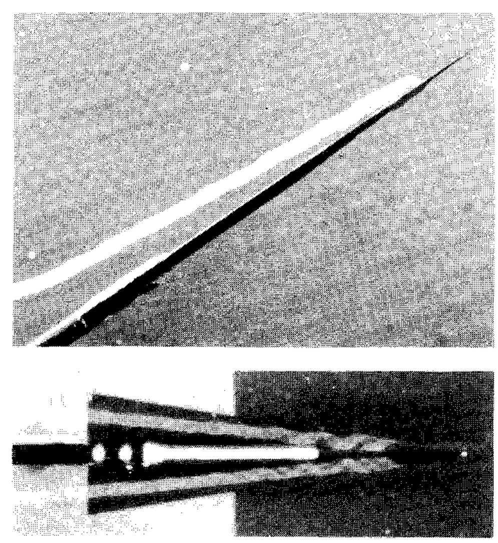
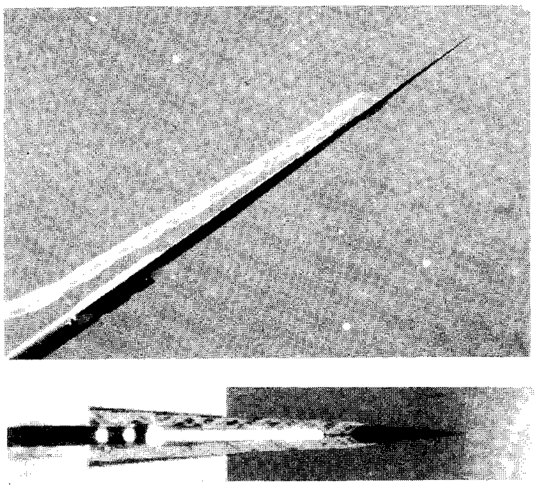
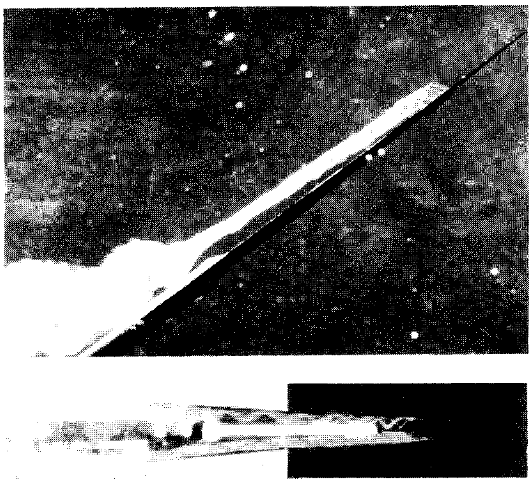


Fig. 3 Vortex flow behind delta wing: $A = 0.56$; $Re_c = 3.4 \times 10^4$; $\alpha = 35$ deg.



a) α = 34.5 deg



b) α = 36 deg

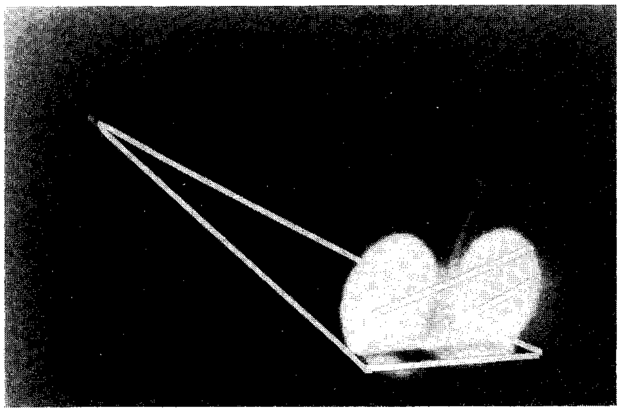
Fig. 4 Vortex flow behind delta wing: A = 0.28; Re_c = 3.4 × 10^4.

Flow-visualization tests carried out on these two delta wings in the water tunnel revealed that on both wings the vortex flow was, at least nominally, symmetric at all incidences before vortex breakdown had moved upstream onto the wing. This result is not consistent with the results reported in the literature.[1-3] Therefore, further tests were carried out on a larger wing model with A = 0.56 in a wind tunnel at a larger Reynolds number. Smoke flow visualizations, again, revealed only symmetric vortex configurations before vortex breakdown reached the wing.

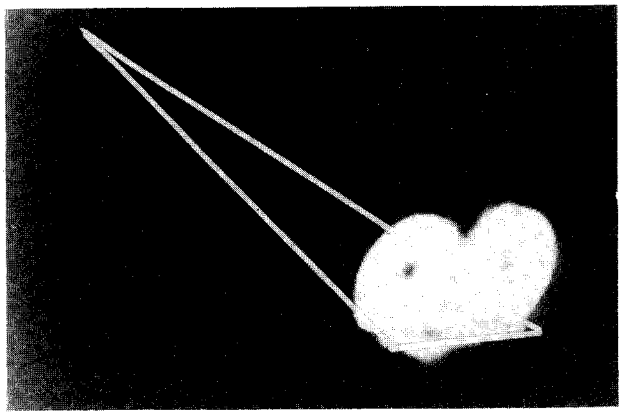
A number of flow-visualization pictures are given in Ref. 7; here, only a few examples are presented. Figure 3 shows the delta wing with aspect ratio A = 0.56 (φ = 82 deg) in the water flow at angle of incidence α = 35 deg. The side view and top view clearly show symmetry of the vortex flow before vortex breakdown reaches the trailing edge (occurring there at $\alpha \cong$ 37 deg). In contrast to this, onset of strong asymmetry is predicted in Fig. 2 at $\alpha_{onset} \cong$ 24 deg. The vortex-flow patterns behind the very slender delta wing with A = 0.28 (φ = 86 deg) are shown in Fig. 4 for angles of incidence α = 34.5 and 36 deg. The vortices are seen to be, at least nominally, symmetric at α = 34.5 deg. At α = 36 deg, vortex breakdown occurs on one side on the wing near the trailing edge. Even at this incidence, the regular, unburst front portion of the vortices are largely symmetric down to the location of onset of vortex breakdown. In contrast to our findings, onset of strong asymmetry is predicted in Fig. 2 at $\alpha \cong$ 16 deg.

The vortex-flow patterns behind the delta wing with A = 0.56, obtained in laser-light sheets by means of smoke in the wind tunnel, are seen in Fig. 5 for angles of incidence 25 deg $\leq \alpha \leq$ 35 deg. The vortex configurations at α = 25 and 30 deg are, at least nominally, symmetric; at α = 35 deg, one vortex has undergone breakdown.

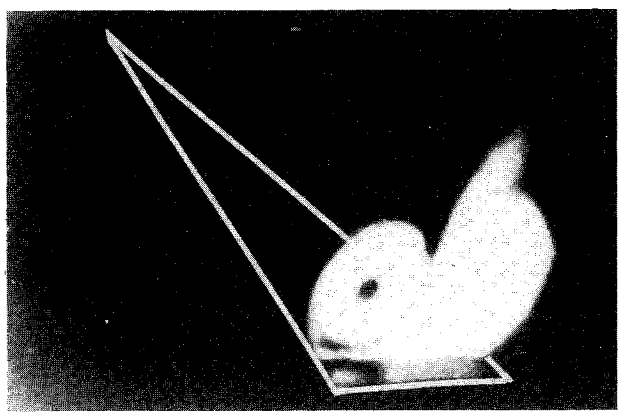
Further experiments were carried out in the wind tunnel with the delta wing (A = 0.56) at one yaw angle β = 6 deg and high incidence α = 30 and 35 deg in order to find out whether this distinct yaw would bring on strong asymmetry of the vortex-flow configuration. Flow visualizations showed that the vortex flow, at α = 30 deg, was only slightly asymmetric in light planes at midchord and near the trailing edge. At α = 35 deg, vortex breakdown was present on the windward portion, near the trailing edge, sometimes reaching for-



a) α = 25 deg



b) α = 30 deg



c) α = 35 deg

Fig. 5 Vortex flow behind delta wing: A = 0.56; Re_c = 1.32 × 10^5.

ward to the middle of the wing; on the opposite side, the vortex flow was regular. On the front part of the wing, the vortex flow was regular on both sides, showing a slight asymmetry, as can be expected. Our tests do not reveal strong asymmetry of the vortex flow on the delta wing with A = 0.56 at high incidence and a distinct yaw angle.

The present results from water-tunnel and wind-tunnel tests have been entered into Fig. 2 where they can be compared with the boundaries for onset of strong vortex-flow asymmetry and vortex breakdown. The angles of incidence for the first occurrence of vortex breakdown on the two wings with A = 0.56 (φ = 82 deg) in the water-tunnel and wind-tunnel tests, respectively, agree fairly well with the data given in Fig. 2 from Ref. 3. No comparison is possible of the present results for the wing with A = 0.28 (φ = 86 deg) due to the lack of data from other sources.

The boundary of strong vortex-flow asymmetry in Fig. 2 was, as it seems, given initially by Polhamus.[1] He reported that "an extremely slender wing phenomenon, such as vortex contact[8] or vortex asymmetry[9] can occur before vortex breakdown" and illustrated the effect of vortex asymmetry on a delta wing with aspect ratio A = 0.25 (φ = 86.5 deg). No information was given as to the detailed geometry of this wing.

In Ref. 8, low-speed flow investigations of the vortex patterns were carried out on three slender flat plates with sharp edges and planforms, as shown in Fig. 6. The plates all produced the same type of flow, and the observations reported on the rectangular plate apply qualitatively to all three. On the rectangular plate of A = 0.083, at incidence α = 25 deg and zero yaw, the vortex sheets rolled up symmetrically on the lee side. When the wing was yawed, the attachment line on the lee surface became inclined toward the trailing side and the vortex on this side was somewhat distorted, but not strongly asymmetric. Only at higher angles of yaw, when the attachment line intersected the edge on the trailing side, did the vortex on this side break away and a strongly asymmetric flow was formed. The change in the flow pattern with increasing yaw is reported to have been gradual, in contrast to the unstable changes on a body of revolution.

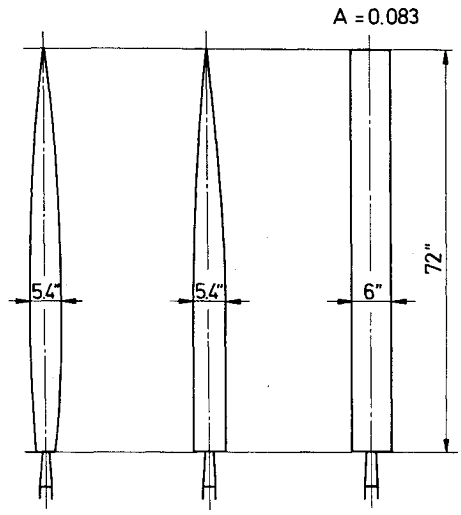


Fig. 6 Geometry of flat, sharp-edged plates studied by Maltby and Peckham.[8]

Bird[9] carried out tuft-grid surveys of the flow past delta wings with various leading-edge sweep angles φ in low-speed flow at Reynolds number $Re \cong 2.8 \times 10^6$ (l = 0.3 m) and incidences up to α = 30 deg at zero yaw. The wings all had flat-plate airfoil sections with rounded leading edges and beveled trailing edges. Lacking respective detailed information, it was assumed that the rounding of the leading edges had resulted in semicircles, or nearly so, in sections normal to the leading edge. Two of the flat-plate wings are shown in Fig. 7 in plan view and various cross sections; in these cross sections, the assumed leading-edge shape is also practically semicircular. On the very slender delta wing, with φ = 86.5 deg, flow observations revealed the onset of strongly asymmetric vortex flow at an incidence of α = 15 deg, with such asymmetry up to the highest incidence α = 30 deg. These results are plotted in Fig. 2 and are seen to be in agreement with the asymmetry boundary. In contrast to this result, the delta wing with leading-edge sweep angle φ = 82.5 deg exhibited on the available tuft-grid survey pictures from α = 15 to 30 deg a nominally symmetrical vortex flow. These results, also included in Fig. 2, do not support the asymmetry boundary.

Apart from experimental evidence, results of theoretical work, pertinent to the phenomenon of asymmetric vortex flow, are available. Fiddes and Williams[6] presented results of a theoretical inviscid-flow study of vortex asymmetry on slender bodies of various cross sections. The free shear layers were represented by a vortex-sheet model; the particular form used was that of Smith,[10] which was developed for leading-edge separation from slender, thin delta wings. The inner turns of the vortex sheet are represented by a line vortex, connected by a cut to the end of the truncated vortex sheet. Together with the results obtained by Smith[11] for the separation of a vortex sheet from a smooth surface, Fiddes[12] developed a complete vortex-sheet model for inviscid flow past

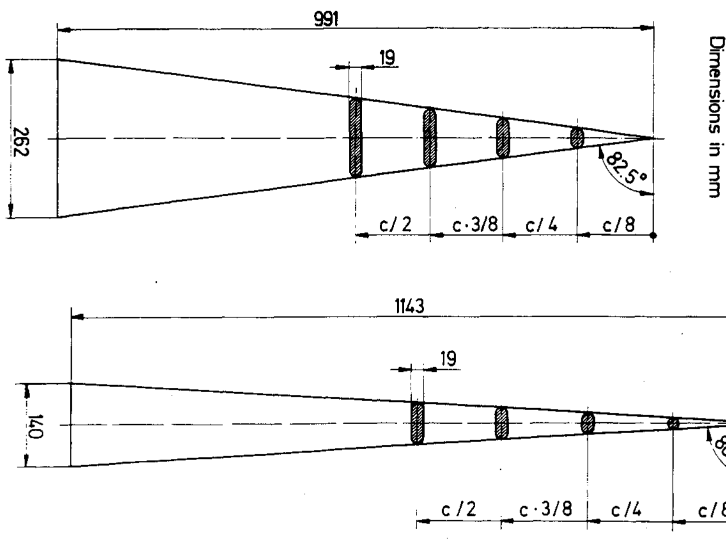


Fig. 7 Delta wing models studied by Bird.[9]

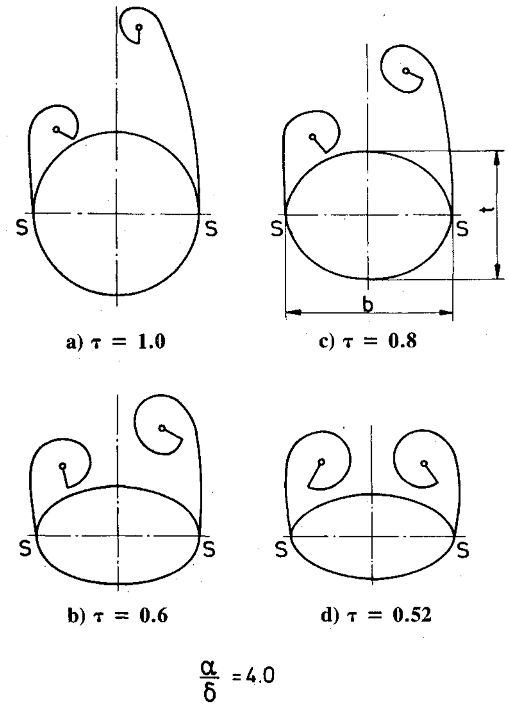


Fig. 8 Effect of cross-sectional thickness ratio $\tau = t/b$ of elliptic cones on vortex asymmetry[6]: δ = semiapex angle; $\alpha/\delta = 4.0$.

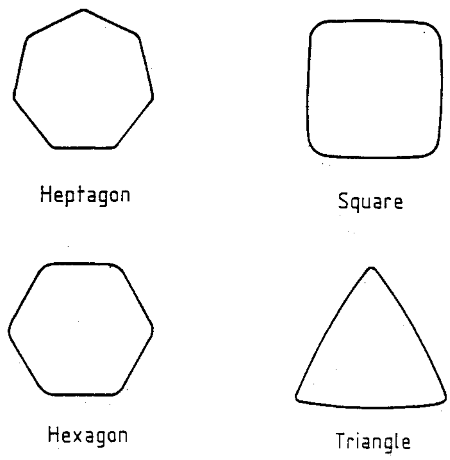


Fig. 9 Polygonal cross-sectional shapes of slender cones.[6]

slender circular and elliptic cones. The sheet was represented by a contiguous set of circular arcs. In this inviscid-flow model, the separation-line positions were specified as a parameter. A Newton-type iteration technique was used to determine the sheet shape and strength.

Solutions were obtained with this vortex-sheet model for the flow past elliptic cones with symmetrical separation-line positions specified. Two families of solutions occurred, one for symmetrical and one for asymmetrical vortex flows. In Ref. 6, results are reported on the effect of the thickness ratio of elliptic cones on vortex-flow asymmetry, with flow separation specified at the ends of the major axes of the ellipses. In Fig. 8, it is seen that the degree of asymmetry of the vortex flow is reduced as the thickness ratio is decreased and at 52% thickness the asymmetry has vanished at the specific incidence chosen. At higher incidences, the degree of flattening needed to suppress asymmetry increases, but at a very slow rate.

This result suggests that on thin elliptic cones, and in the limit on delta wings, vortex-flow asymmetry may not be found. This is confirmed by results of studies using a line-vortex model; they indicate that asymmetric solutions do not occur at any incidence on a delta wing. This, together with the vortex-sheet results, strongly suggests that asymmetric vortex flows, of the type found on elliptic cones, do not occur on delta wings.

The method described by Fiddes[12] can be used for slender cones of arbitrary cross-sectional shapes (as long as a shape can be mapped to a circle and the boundary condition at the body can be enforced). Examples of cross-sectional shapes studied (polygons) are shown in Fig. 9. Typical results obtained for the vortex-flow configurations on slender cones with two different cross-sectional shapes are shown in Fig. 10, separation being prescribed at the shoulders of the bodies. The effect of change of cross-sectional shape going away from the circle is to reduce the degree of vortex-flow asymmetry.

Discussion

The results obtained by Bird,[9] showing distinctly asymmetric vortex flow on his delta wing with leading-edge sweep angle φ = 86.5 deg, are in contrast with our results for a delta wing with φ = 86 deg, having symmetric vortex flow up to vortex breakdown on the wing. There are certainly differences between the test conditions of the two studies: e.g., Reynolds number, turbulence levels and structure, and noise environment, but also in model geometry. The cross-sectional shapes of the two wings, at various distances x from the apex in the forward part, are shown in Fig. 11. There, our wings's local span has been drawn for comparison purposes with the same width as that of Bird's wing. Also indicated are the thickness ratios τ of the local cross sections.

Examining the cross-sectional shapes of Bird's model (as were conjectured on the basis of the incomplete information available), one notices that the front part of the wing, from the apex as far back as

$$x \cong \tfrac{1}{4} c$$

resembles more a distorted, thick elliptic cone, with probably a circular cone near the tip, than a thin flat-plate wing. This could suggest that the front part is prone to exhibit vortex-flow asymmetry. It is known that on slender pointed noses the flow past the entire nose with regard to symmetry or asymmetry is influenced by conditions at the tip (nose blunting, blowing, addition of grits). The results of the theoretical study by Fiddes and Williams[6] indicate that slender elliptic cones exhibit vortex-flow asymmetry above a certain thickness ratio, with an increasing degree of asymmetry for increasing thickness.

Comparing in Fig. 11 the cross-sectional shapes of Bird's wing with that of our wing in the front part reveals a distinct

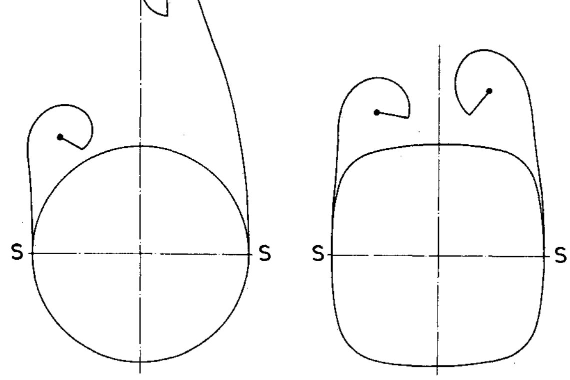
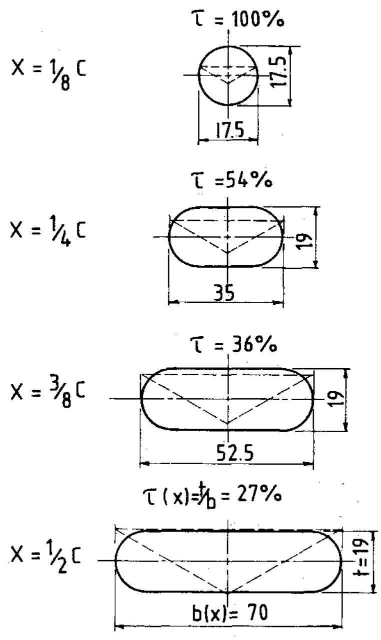


Fig. 10 Effect of variation of cross-sectional shape of slender cone on vortex asymmetry[6]; δ = semiapex angle; $\alpha/\delta = 4.1$.



Delta wing	φ deg.	τ (x)	Cross section
J. D. Bird Ref. 9	86.5	Varying	Flat plate rounded
Present	86	29%	Triangular

Fig. 11 Cross-sectional shapes and thickness ratio τ for two delta wings.

difference between Bird's rounded, more or less elliptical shapes and our thinner, triangular shape. The results of Fiddes and Williams[6] as to the effect of change of cross-sectional shape indicate that going away from the round shape to polygons with decreasing number of sides reduces the degree of asymmetry of vortex flow on slender cones, as was shown in Figs. 9 and 10.

We compare now the two delta wings of Bird[9] with leading-edge sweep angles of $\varphi = 86.5$ and 82.5 deg, respectively. The very slender wing ($\varphi = 86.5$ deg) exhibits, at incidences 15 deg $\leq \alpha \leq 30$ deg, a strongly asymmetric vortex flow, in contrast with the other, less slender wing ($\varphi = 82.5$ deg), which has, at least nominally, a symmetric vortex configuration for incidences 15 deg $\leq \alpha \leq 30$ deg. Again, the assumption is made for the very slender wing that the front part has cross-sectional shapes somewhat like an elliptic cone, becoming circular near the apex (see Fig. 7). Then, we have near the tips circular cones of semiapex angles $\delta_N = 3.5$ deg ($\varphi = 86.5$ deg) and $\delta_N = 7.5$ deg ($\varphi = 82.5$ deg). It is known that on pointed forebodies the angle of incidence for onset of vortex-flow asymmetry increases practically proportionally with the apex angle.[13] Applying this result to the present situation, it seems possible that the onset of vortex-flow asymmetry on the $\varphi = 82.5$ deg wing will occur somewhat above the highest incidence tested ($\alpha = 30$ deg), for which the latter still exhibits symmetric flow.

Conclusions

Flow observations carried out on two slender, sharp-edged delta wings in a water tunnel at high incidence revealed that the vortex flow on the lee side remained symmetrical until vortex breakdown reached the wing. This result was confirmed when testing one of the wing geometries in a wind tunnel at a larger Reynolds number. This is in contrast to the boundary for strong vortex-flow asymmetry reported in the literature. Setting the wing in the wind tunnel at a distinct yaw angle, at high incidence, brought on only slight vortex-flow asymmetry. Subsequent studies of experimental evidence reported for slender flat plates of various planforms at high incidence did not reveal vortex-flow asymmetry either. The one delta wing found exhibiting strong vortex-flow asymmetry probably had near the apex the shape of a more or less thick elliptic cone. This suggests that for a delta wing the geometry near the apex influences the occurrence of vortex-flow asymmetry. Recent results of an inviscid-flow theory for slender cones strongly suggest that on thin, slender delta wings no vortex-flow asymmetry occurs.

It was brought to our attention by the reviewer that similar results have been reported by Ng and Malcolm.[14]

Acknowledgments

The principal author gratefully appreciates having had the opportunity to work at the Institute for Experimental Fluid Mechanics of DLR, Göttingen, summer 1989, and having used its facilities. We all appreciate the assistance of the respective staff at DLR and KFUPM, as well as helpful comments by the reviewer.

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